

**EXERCISE**

1. If  $\tan \theta = 1$ , then find the value of  $\frac{8 \sin \theta + 5 \sin \theta}{\sin^3 \theta - 2 \cos^3 \theta + 7 \cos \theta}$ 
  - (a) 2
  - (b)  $2\frac{1}{2}$
  - (c) 3
  - (d)  $\frac{4}{5}$
2. If  $\theta$  be a positive acute angle satisfying  $\cos^2 \theta + \cos^4 \theta = 1$ , then the value of  $\tan^2 \theta + \tan^4 \theta$  is
  - (a)  $\frac{3}{2}$
  - (b) 1
  - (c)  $\frac{1}{2}$
  - (d) 0
3. The value of  $\tan 4^\circ \tan 43^\circ \tan 47^\circ \tan 86^\circ$  is
  - (a) 0
  - (b) 1
  - (c)  $\sqrt{3}$
  - (d)  $\frac{1}{\sqrt{3}}$
4. If  $\tan 15^\circ = 2 - \sqrt{3}$ , the value of  $\tan 15^\circ \cot 75^\circ + \tan 75^\circ \cot 15^\circ$  is
  - (a) 14
  - (b) 12
  - (c) 10
  - (d) 8
5. The value of  $(\sin^2 1^\circ + \sin^2 3^\circ + \sin^2 5^\circ + \dots + \sin^2 85^\circ + \sin^2 87^\circ + \sin^2 89^\circ)$ 
  - (a)  $21\frac{1}{2}$
  - (b) 22
  - (c)  $22\frac{1}{2}$
  - (d)  $23\frac{1}{2}$
6. If  $\sin \theta - \cos \theta = \frac{7}{13}$  and  $0 < \theta < 90^\circ$ , then the value of  $\sin \theta + \cos \theta$  is.
  - (a)  $\frac{17}{13}$
  - (b)  $\frac{13}{17}$
  - (c)  $\frac{1}{13}$
  - (d)  $\frac{1}{17}$
7. The minimum value of  $\cos 2\theta + \cos \theta$  for real values of  $\theta$  is-
  - (a)  $-\frac{9}{8}$
  - (b) 0
  - (c) -2
  - (d) none of these
8. If  $5 \tan \theta - 4 = 0$ , then the value of  $\frac{5 \sin \theta - 4 \cos \theta}{5 \sin \theta + 4 \cos \theta} = ?$ 
  - (a)  $\frac{5}{3}$
  - (b)  $\frac{5}{6}$
  - (c) 0
  - (d)  $\frac{1}{6}$
9. The value of  $\tan^6 20^\circ - 33 \tan^4 20^\circ + 27 \tan^2 20^\circ$  is:
  - (a) 2
  - (b) 3
  - (c) 4
  - (d) 5
10. If  $\tan \theta = \frac{1}{\sqrt{7}}$ , then  $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} = ?$ 
  - (a)  $\frac{5}{7}$
  - (b)  $\frac{3}{7}$
  - (c)  $\frac{1}{12}$
  - (d)  $\frac{3}{4}$
11. If  $y = \frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha}$  then  $\frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha}$  is equal to
  - (a)  $\frac{1}{y}$
  - (b) y
  - (c)  $1 - y$
  - (d)  $1 + y$
12. A person, standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank is  $60^\circ$ ; when he retreats 20m from the bank, he finds the angle to be  $30^\circ$ . The height of the tree and the breadth of the river are-
  - (a)  $10\sqrt{3}$ m, 10m
  - (b) 10;  $10\sqrt{3}$ m
  - (c) 20m, 30m
  - (d) none of these
13. If  $\theta$  is an acute angle such that  $\tan^2 \theta = \frac{8}{7}$ , then the value of  $\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$  is
  - (a)  $\frac{7}{8}$
  - (b)  $\frac{8}{7}$
  - (c)  $\frac{7}{4}$
  - (d)  $\frac{64}{49}$
14. If  $3 \cos \theta = 5 \sin \theta$ , then the value of  $\frac{5 \sin \theta - 2 \sec^3 \theta + 2 \cos \theta}{5 \sin \theta + 2 \sec^3 \theta - 2 \cos \theta}$  is equal to
  - (a)  $\frac{271}{979}$
  - (b)  $\frac{376}{2937}$
  - (c)  $\frac{542}{2937}$
  - (d) none of these
15. If  $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$  and  $x \sin \theta = y \cos \theta$  then  $x^2 + y^2 =$ 
  - (a) 1
  - (b) 2
  - (c) 0
  - (d) None

16. If  $1 + \sin^2 A = 3 \sin A \cos A$ , then what are the possible values of  $\tan A$ ?  
 (a)  $\frac{1}{4}, 2$  (b)  $\frac{1}{6}, 3$   
 (c)  $\frac{1}{2}, 1$  (d)  $\frac{1}{8}, 4$
17. The value of  $\frac{\cos^3 20^\circ - \cos^3 70^\circ}{\sin^3 70^\circ - \sin^3 20^\circ}$  is  
 (a)  $\frac{1}{2}$  (b)  $\frac{1}{\sqrt{2}}$   
 (c) 1 (d) 2
18. If  $\frac{x \operatorname{cosec}^2 30^\circ \cdot \sec^2 45^\circ}{8 \cos^2 45^\circ \cdot \sin^2 60^\circ} = \tan^2 60^\circ - \tan^2 30^\circ$ , then  $x = ?$   
 (a) 1 (b) -1  
 (c) 2 (d) 0
19. If  $\theta + \phi = \frac{\pi}{6}$ , what is the value of  $(\sqrt{3} + \tan \theta)(3 + \tan \phi)$ ?  
 (a) 1 (b) -1  
 (c) 4 (d) -4
20. If  $\theta$  is an acute angle such that  $\sec^2 \theta = 3$ , then  $\frac{\tan^2 \theta - \operatorname{cosec}^2 \theta}{\tan^2 \theta + \operatorname{cosec}^2 \theta}$  is  
 (a)  $\frac{4}{7}$  (b)  $\frac{3}{7}$   
 (c)  $\frac{2}{7}$  (d)  $\frac{1}{7}$
21. What should be the height of a flag where a 20 feet long ladder reaches 20 feet below the flag (The angle of elevation of the top of the flag at the foot of the ladder is  $60^\circ$ )?  
 (a) 20 feet (b) 30 feet  
 (c) 40 feet (d)  $20\sqrt{2}$  feet
22. If  $\theta$  &  $2\theta - 45^\circ$  are acute angles such that  $\sin \theta = \cos(2\theta - 45^\circ)$  then  $\tan \theta$  is equal to  
 (a) 1 (b) -1  
 (c)  $\sqrt{3}$  (d)  $\frac{1}{\sqrt{3}}$
23. If  $5\theta$  &  $4\theta$  are acute angles satisfying  $\sin 5\theta = \cos 4\theta$  then  $2 \sin 3\theta - \sqrt{3} \tan 3\theta = ?$   
 (a) 1 (b) 0  
 (c) -1 (d)  $\frac{1}{\sqrt{3}}$
24. A vertical pole with height more than 100 m consists of two parts, the lower being one-third of the whole. At a point on a horizontal plane through the foot and 40 m from it, the upper part subtends an angle whose tangent is  $\frac{1}{2}$ . What is the height of the pole?  
 (a) 110m (b) 200m  
 (c) 120m (d) 150m
25. If  $\sec \theta + \tan \theta = x$ , then  $\sec \theta = ?$   
 (a)  $\frac{x^2+1}{x}$  (b)  $\frac{x^2+1}{2x}$   
 (c)  $\frac{x^2-1}{2x}$  (d)  $\frac{x^2-1}{x}$
26. The correct value of the parameter 't' of the identity  $2(\sin^6 x + \cos^6 x) + t(\sin^4 x + \cos^4 x) = -1$  is:  
 (a) 0 (b) -1  
 (c) -2 (d) -3
27. If  $a \cos \theta - b \sin \theta = c$ , then  $a \cos \theta + b \sin \theta = ?$   
 (a)  $\pm \sqrt{a^2 + b^2 + c^2}$  (b)  $\pm \sqrt{a^2 + b^2 - c^2}$   
 (c)  $\pm \sqrt{c^2 - a^2 - b^2}$  (d) None of these
28.  $\frac{\tan \theta}{\sec \theta - 1} + \frac{\tan \theta}{\sec \theta + 1}$  is equal to  
 (a)  $2 \tan \theta$  (b)  $2 \sec \theta$   
 (c)  $2 \operatorname{cosec} \theta$  (d)  $2 \tan \theta \cdot \sec \theta$
29. If  $a \cos \theta + b \sin \theta = m$  and  $a \sin \theta - b \cos \theta = n$ , then  $a^2 + b^2 =$   
 (a)  $m^2 - n^2$  (b)  $m^2 n^2$   
 (c)  $n^2 - m^2$  (d)  $m^2 + n^2$
30. The angular elevation of a tower CD at a place A due south of it is  $60^\circ$ ; and at a place B due west of A, the elevation is  $30^\circ$ . If  $AB = 3$  km, the height of the tower is  
 (a)  $2\sqrt{3}$  km (b)  $3\sqrt{6}$  km  
 (c)  $\frac{3\sqrt{3}}{2}$  km (d)  $\frac{3\sqrt{6}}{4}$  km
31. If  $\sin \alpha + \cos \beta = 2$  ( $0^\circ \leq \beta < \alpha \leq 90^\circ$ ), then  $\sin\left(\frac{2\alpha + \beta}{3}\right) =$   
 (a)  $\sin \frac{\alpha}{2}$  (b)  $\cos \frac{\alpha}{3}$   
 (c)  $\sin \frac{\alpha}{3}$  (d)  $\cos \frac{2\alpha}{3}$
32. If  $\cos^4 \theta - \sin^4 \theta = \frac{2}{3}$ , then the value of  $2 \cos^2 \theta - 1$  is  
 (a) 0 (b) 1  
 (c)  $\frac{2}{3}$  (d)  $\frac{3}{2}$

33. If  $\sin \alpha \sec (30^\circ + \alpha) = 1$  ( $0 < \alpha < 60^\circ$ ), then the value of  $\sin \alpha + \cos 2\alpha$  is  
 (a) 1 (b)  $\frac{2+\sqrt{3}}{2\sqrt{3}}$   
 (c) 0 (d)  $\sqrt{2}$
34. The minimum value of  $2 \sin^2 \theta + 3 \cos^2 \theta$  is  
 (a) 0 (b) 3  
 (c) 2 (d) 1
35. If  $\operatorname{cosec} 39^\circ = x$ , the value of  $\frac{1}{\operatorname{cosec}^2 51^\circ} + \sin^2 39^\circ + \tan^2 51^\circ = \frac{1}{\sin^2 51^\circ \sec^2 39^\circ}$  is  
 (a)  $\sqrt{x^2 - 1}$  (b)  $\sqrt{1 - x^2}$   
 (c)  $x^2 - 1$  (d)  $1 - x^2$
36. If  $A = \sin^2 \theta + \cos^4 \theta$ , for any value of  $\theta$ , then the value of A is  
 (a)  $1 \leq A \leq 2$  (b)  $\frac{3}{4} \leq A \leq 2$   
 (c)  $\frac{13}{16} \leq A \leq 1$  (d)  $\frac{3}{4} \leq A \leq \frac{13}{16}$
37. If  $\tan 2\theta, \tan 4\theta = 1$ , then the value of  $\tan 3\theta$  is  
 (a)  $\sqrt{3}$  (b) 0  
 (c) 1 (d)  $\frac{1}{\sqrt{3}}$
38. If  $(\theta_1 + \theta_2) = \sqrt{3}$  and  $\sec (\theta_1 - \theta_2) = \frac{2}{\sqrt{3}}$ , then the value of  $\sin 2\theta_1 + \tan 3\theta_2$  is equal to (assume that  $0 < \theta_1 - \theta_2 < \theta_1 + \theta_2 < 90^\circ$ )  
 (a) 0 (b) 3  
 (c) 1 (d) 2
39. If  $\sec \theta = x + \frac{1}{4x}$  ( $0^\circ < \theta < 90^\circ$ ), then  $\sec \theta + \tan \theta$  is equal to  
 (a)  $\frac{x}{2}$  (b)  $2x$   
 (c)  $x$  (d)  $\frac{1}{2x}$
40. If  $x = a \sec \theta \cos \phi$ ,  $y = b \sec \phi$ ,  $z = c \tan \theta$ , then, the value of  $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$  is:  
 (a) 1 (b) 4  
 (c) 9 (d) 0
41. If  $\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = \frac{5}{3}$ , then  $\sin \theta$  is equal to:  
 (a)  $\frac{1}{4}$  (b)  $\frac{1}{3}$   
 (c)  $\frac{2}{3}$  (d)  $\frac{3}{4}$
42. If  $0 \leq \theta \leq \frac{\pi}{2}$  and  $\sec^2 \theta + \tan^2 \theta = 7$ , then  $\theta$  is  
 (a)  $\frac{5\pi}{12}$  radian (b)  $\frac{\pi}{3}$  radian  
 (c)  $\frac{\pi}{5}$  radian (d)  $\frac{\pi}{6}$  radian
43. The simplest value of  $\sin^2 x + 2 \tan^2 x - 2 \sec^2 x + \cos^2 x$  is  
 (a) 1 (b) 0  
 (c) -1 (d) 2
44. A kite is flying at a height of 50 metre. If the length of string is 100 metre then the inclination of string to the horizontal ground in degree measure is  
 (a) 90 (b) 60  
 (c) 45 (d) 30
45. From the top of a light-house at a height 20 metres above sea-level, the angle of depression of a ship is  $30^\circ$ . The distance of the ship from the foot of the light-house is  
 (a) 20m (b)  $20\sqrt{3}$ m  
 (c) 30m (d)  $30\sqrt{3}$ m
46. If  $x = a \sin \theta$  and  $y = b$  then prove that  $\frac{a^2}{x^2} - \frac{b^2}{y^2}$  is  
 (a) 1 (b) 2  
 (c) 3 (d) 4
47. If  $2y \cos \theta = x \sin \theta$  and  $2x \sec \theta - y \operatorname{cosec} \theta = 3$ , then the relation between x and y is  
 (a)  $2x^2 + y^2 = 2$  (b)  $x^2 + 4y^2 = 4$   
 (c)  $x^2 + 4y^2 = 1$  (d)  $4x^2 + y^2 = 4$
48. If  $\sec \theta + \tan \theta = \sqrt{3}$ , then the positive value of  $\sin \theta$  is  
 (a) 0 (b)  $\frac{1}{2}$   
 (c)  $\frac{\sqrt{3}}{2}$  (d) 1
49. The radian measure of  $63^\circ 14' 15''$  is  
 (a)  $\left(\frac{2811\pi}{8000}\right)^c$  (b)  $\left(\frac{3811\pi}{8000}\right)^c$   
 (c)  $\left(\frac{4811\pi}{8000}\right)^c$  (d)  $\left(\frac{5811\pi}{8000}\right)^c$
50. If  $\frac{\cos^4 \alpha}{\cos^2 \beta} + \frac{\sin^4 \alpha}{\sin^2 \beta} = 1$ , then the value of  $\frac{\cos^4 \beta}{\cos^2 \alpha} + \frac{\sin^4 \beta}{\sin^2 \alpha}$  is  
 (a) 4 (b) 0

- (c)  $\frac{1}{8}$  (d) 1
51.  $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$  (where  $\theta \neq \frac{\pi}{2}$ ) is equal to  
 (a)  $\frac{1 + \sin\theta}{\cos\theta}$  (b)  $\frac{1 - \sin\theta}{\cos\theta}$   
 (c)  $\frac{1 - \cos\theta}{\sin\theta}$  (d)  $\frac{1 + \cos\theta}{\sin\theta}$
52. The angles of elevation of the top of a tower standing on a horizontal plane from two points on a line passing through the foot of the tower at a distance 9 ft and 16 ft respectively are complementary angles. Then the height of the tower is  
 (a) 9 ft (b) 12 ft  
 (c) 16 ft (d) 144 ft
53. If  $\sin^2\alpha = \cos^3\alpha$ , then the value of  $(\cot^6\alpha - \cot^2\alpha)$  is  
 (a) 1 (b) 0  
 (c) -1 (d) 2
54. The simplified value of  $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$  is  
 (a) -2 (b) 2  
 (c) 1 (d) -1
55. The value of  $\frac{\sin 53^\circ}{\cos 37^\circ} \div \frac{\cot 65^\circ}{\tan 25^\circ}$  is  
 (a) 2 (b) 1  
 (c) 3 (d) 0
56. The value of  $\frac{\cos 60^\circ + \sin 60^\circ}{\cos 60^\circ - \sin 60^\circ}$  is  
 (a) -1 (b)  $\sqrt{3} + 2$   
 (c)  $-(2 + \sqrt{3})$  (d)  $\sqrt{3} - 2$
57. The value of  $\frac{\cot 5^\circ \cdot \cot 10^\circ \cdot \cot 15^\circ \cdot \cot 60^\circ \cdot \cot 75^\circ \cdot \cot 80^\circ \cdot \cot 85^\circ}{(\cos^2 20^\circ + \cos^2 70^\circ) + 2}$  is  
 (a)  $\frac{9}{\sqrt{3}}$  (b)  $\frac{1}{9}$   
 (c)  $\frac{1}{\sqrt{3}}$  (d)  $\frac{\sqrt{3}}{9}$
58. In a triangle, the angles are in the ratio 2:5:3. What is the value of the least angle in the radian?  
 (a)  $\frac{\pi}{20}$  (b)  $\frac{\pi}{10}$   
 (b)  $\frac{2\pi}{5}$  (d)  $\frac{\pi}{5}$
59. If  $x = a \cos\theta - b \sin\theta$ ,  $y = b \cos\theta + a \sin\theta$ , then the find the value of  $x^2 + y^2$ .  
 (a)  $a^2$  (b)  $b^2$   
 (c)  $\frac{a^2}{b^2}$  (d)  $a^2 + b^2$
60. If  $\tan\alpha + \cot\alpha = 2$ , then the value of  $\tan^7\alpha + \cot^7\alpha$  is  
 (a) 2 (b) 16  
 (b) 64 (d) 128
61. From 125 metre high towers, the angle of depression of a car is  $45^\circ$ . Then how far the car is from the tower?  
 (a) 125 metre (B) 60 metre  
 (b) 75 metre (d) 95 metre
62. value of  $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$  is a equal to  
 (a) -1 (b) 1  
 (b) 2 (d) 0
63. The shadow of a tower standing on a level plane is found to be 30 m longer when the Sun's altitude changes from  $60^\circ$  to  $45^\circ$ . The height of the tower is  
 (a)  $15(3 + \sqrt{3})$ m (b)  $15(\sqrt{3} + 1)$ m  
 (b)  $15(\sqrt{3} - 1)$ m (d)  $15(3 - \sqrt{3})$ m
64. If  $\sin 17^\circ = \frac{x}{y}$ . Then  $\sec 17^\circ - \sin 73^\circ$  is equal to  
 (a)  $\frac{y}{\sqrt{y^2 - x^2}}$  (b)  $\frac{y}{(x\sqrt{y^2 - x^2})}$   
 (b)  $\frac{y}{(y\sqrt{y^2 - x^2})}$  (d)  $\frac{x^2}{(y\sqrt{y^2 - x^2})}$
65. If  $\theta$  is a positive acute angle and  $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$ , then the value of  $\operatorname{cosec}\theta$  is  
 (a)  $\frac{1}{\sqrt{3}}$  (b)  $\sqrt{3}$   
 (c)  $\frac{2}{\sqrt{3}}$  (d) 1
66. If  $\cos\alpha + \sec\alpha = \sqrt{3}$ , then the value of  $\cos^3\alpha + \sin^3\alpha$  is  
 (a) 2 (b) 1  
 (b) 0 (d) 4

67. If  $\sin \theta + \cos \theta = \sqrt{2} \cos \theta$ , then the value of  $\cot \theta$  is  
 (a)  $\sqrt{2} + 1$  (b)  $\sqrt{2} - 1$   
 (c)  $\sqrt{3} - 1$  (d)  $\sqrt{3} + 1$
68. The value of  $\sin^2 1^\circ + \sin^2 2^\circ + \sin^2 3^\circ + \dots + \sin^2 89^\circ$  is  
 (a) 22 (b) 44  
 (c)  $22\frac{1}{2}$  (d)  $44\frac{1}{2}$

**ANSWER KEY**

1.(a)	2.(b)	3.(b)	4.(a)	5.(c)
6.(a)	7.(a)	8.(c)	9.(b)	10.(d)
11.(b)	12.(a)	13.(a)	14.(a)	15.(a)
16.(c)	17.(c)	18.(a)	19.(c)	20.(d)
21.(b)	22.(a)	23.(b)	24.(c)	25.(b)
26.(d)	27.(b)	28.(c)	29.(d)	30.(d)
31.(b)	32.(c)	33.(a)	34.(b)	35.(c)
36.(b)	37.(c)	38.(d)	39.(b)	40.(a)
41.(a)	42.(b)	43.(c)	44.(d)	45.(b)
46.(a)	47.(b)	48.(b)	49.(a)	50.(d)
51.(a)	52.(b)	53.(a)	54.(b)	55.(b)
56.(c)	57.(d)	58.(d)	59.(d)	60.(a)
61.(a)	62.(c)	63.(a)	64.(d)	65.(c)
66.(c)	67.(a)	68.(d)		

**HINTS & EXPLANATIONS**

1.  $\tan \theta = 1$   
 $\sec \theta = \sqrt{1 + \tan^2 \theta}$   
 $= \sqrt{1 + 1}$   
 $= \sqrt{2}$   
 $\tan \theta = \frac{1}{\sqrt{2}}$   
 $\sec = \sqrt{1 - \cos^2 \theta}$   
 $= \sqrt{1 - \left[\frac{1}{\sqrt{2}}\right]^2} = \frac{1}{\sqrt{2}}$   
 Now,  $\frac{8\sin \theta + 5\sin \theta}{\sin^3 \theta - 2\cos^3 \theta + 7\cos \theta}$   
 $\frac{8X\frac{1}{\sqrt{2}} + 5X\frac{1}{\sqrt{2}}}{\left[\frac{1}{\sqrt{2}}\right]^3 - 2X\left[\frac{1}{\sqrt{2}}\right]^3 + 7X\frac{1}{\sqrt{2}}}$   
 $= \frac{\frac{8+5}{\sqrt{2}}}{\frac{1-2+14}{2\sqrt{2}}} = \frac{(8+5)\sqrt{2}}{1-2+14}$   
 $= \frac{13\sqrt{2}}{13} = 2$
2. (b) Given,  $\cos^2 \theta + \cos^4 \theta = 1$   
 Or  $\cos^4 \theta = 1 - \cos^2 \theta$  [ $\because \sin^2 \theta + \cos^2 \theta = 1$ ]  
 $\cos^4 \theta = \sin^2 \theta$   
 Or,  $1 = \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{1}{\cos^2 \theta} \Rightarrow \tan^2 \theta \cdot \sec^2 \theta = 1$   
 $\tan^2 \theta (1 + \tan^2 \theta) = 1$   
 $[\because \sec^2 \theta - \tan^2 \theta = 1]$   
 $\Rightarrow \tan^2 \theta + \tan^4 \theta = 1$
3. (b)  $\tan 4^\circ \cdot \tan 43^\circ \cdot \tan 47^\circ \cdot \tan 86^\circ$   
 $= \tan 4^\circ \cdot \tan 43^\circ \cdot \tan(90^\circ - 43^\circ) \cdot \tan(90^\circ - 4^\circ)$   
 $= \tan 4^\circ \cdot \tan 43^\circ \cdot \cot 43^\circ \cdot \cot 4^\circ$   
 $[\because \tan(90^\circ - \theta) = \cot \theta]$   
 $= \tan 4^\circ \times \tan 43^\circ \times \frac{1}{\tan 43^\circ} \times \frac{1}{\tan 4^\circ}$   
 $[\because \cot \theta = \frac{1}{\tan \theta}]$   
 $= 1.$
4. (a)  $\tan 15^\circ \cdot \cot 75^\circ \cdot \tan 75^\circ \cdot \cot 15^\circ$   
 $= \tan 15^\circ \cdot \cot(90^\circ - 15^\circ) + \tan(90^\circ - 15^\circ) \cdot \cot 15^\circ$   
 $= \tan 15^\circ \cdot \tan 15^\circ + \cot 15^\circ \cdot \cot 15^\circ$   
 $= (\tan 15^\circ)^2 + (\cot 15^\circ)^2$

$$= (\tan 15^\circ)^2 + \frac{1}{(\tan 15^\circ)^2}$$

Putting the value of  $\tan 15^\circ = 2 - \sqrt{3}$

$$= (2 - \sqrt{3})^2 + \left[ \frac{1}{(2 - \sqrt{3})} \right]^2$$

$$= (2 - \sqrt{3})^2 + \left[ \frac{1}{(2 - \sqrt{3})} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \right]^2$$

$$= (2 - \sqrt{3})^2 + \left[ \frac{2 + \sqrt{3}}{4 - 3} \right]^2$$

$$= (2 - \sqrt{3})^2 + (2 - \sqrt{3})^2$$

$$= 2[2^2 + (\sqrt{3})^2]$$

$$[\because (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)]$$

$$= 2(4 + 3) = 2 \times 7 = 14$$

5. (c) To find total number of terms

First term = 1, last term = 89, common diff = 2

$$a_n = a_1 + (n - 1)d$$

$$89 = 1 + (n - 1) \cdot 2$$

$$\Rightarrow 88 = (n - 1) \cdot 2$$

$$\Rightarrow n - 1 = 44$$

$$\Rightarrow 45 \text{ terms}$$

$$\text{Now, } \sin^2 1^\circ + \sin^2 3^\circ + \sin^2 5^\circ + \dots + \sin^2 85^\circ + \sin^2 87^\circ + \sin^2 89^\circ$$

$$= (\sin^2 1^\circ + \sin^2 89^\circ) + (\sin^2 3^\circ + \sin^2 87^\circ) + \dots + 22 \text{ terms} + \sin^2 45^\circ$$

$$= (\sin^2 1^\circ + \cos^2 1^\circ) + (\sin^2 3^\circ + \cos^2 3^\circ) + \dots + 22$$

$$\text{terms} + \left[ \frac{1}{\sqrt{2}} \right]^2$$

$$= (1 + 1 + \dots + 22 \text{ terms}) + \frac{1}{2}$$

$$= 22 + \frac{1}{2} = 22 \frac{1}{2}$$

6. (a)  $(\sin \theta + \sin \theta)^2 + (\sin \theta - \cos \theta)^2$

$$= (\sin^2 \theta + \cos^2 \theta) + 2 \sin \theta \cdot \cos \theta + (\sin^2 \theta + \cos^2 \theta) - 2 \sin \theta \cdot \cos \theta$$

$$= 1 + 1 = 2$$

$$\text{So, } (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$$

$$\text{Or } (\sin \theta + \cos \theta)^2 + \left[ \frac{7}{13} \right]^2 = 2$$

$$\text{Or, } (\sin \theta + \cos \theta)^2 = 2 - \frac{49}{169} = \frac{289}{169}$$

$$\sin \theta + \cos \theta = \sqrt{\left[ \frac{17}{13} \right]^2} = \frac{17}{13}$$

7. (a) Let  $S = \cos 2\theta + \cos \theta = 2 \cos^2 \theta - 1 + \cos \theta$

$$= -1 + 2 \left[ \cos^2 \theta + \frac{1}{2} \cos \theta + \frac{1}{16} \right] - \frac{1}{8}$$

$$= -\frac{9}{8} + 2 \left[ \cos \theta + \frac{1}{4} \right]^2 \geq -\frac{9}{8}$$

So, the minimum value  $S = -9/8$

8. (c) Given,  $5 \tan \theta - 4 = 0$

$$\Rightarrow \tan \theta = \frac{4}{5}$$

$$\text{Expression, } \frac{(5 \sin \theta - 4 \cos \theta) \cos \theta}{(5 \sin \theta + 4 \cos \theta) \cos \theta} = \frac{5 \tan \theta - 4}{5 \tan \theta + 4}$$

$$= \frac{5 \times \frac{4}{5} - 4}{5 \times \frac{4}{5} + 4}$$

$$= \frac{4 - 4}{4 + 4} = \frac{0}{8} = 0$$

9. (b)  $\sqrt{3} = \tan 60^\circ = \tan(3 \times 20^\circ) = \frac{3 \tan 20^\circ - \tan^3 20^\circ}{1 - 3 \tan^2 20^\circ}$

$$\text{Squaring, } 3 = \frac{9t^2 + t^6 - 6t^4}{1 + 9t^4 - 6t^2}, \tan 20^\circ = t$$

$$\Rightarrow t^6 - 33t^4 + 27t^2 = 3$$

$$\Rightarrow \tan^6 20^\circ - 33 \tan^4 20^\circ + 27 \tan^2 20^\circ = 3$$

10. (d) Given,  $\tan \theta = \frac{1}{\sqrt{7}}$

$$\sec \theta = \sqrt{1 + \tan^2 \theta} = \sqrt{1 + \left( \frac{1}{\sqrt{7}} \right)^2} = \sqrt{\frac{8}{7}}$$

$$\operatorname{cosec} \theta = \frac{\sec \theta}{\tan \theta} = \frac{\sqrt{\frac{8}{7}}}{\frac{1}{\sqrt{7}}} = \sqrt{8}$$

$$\therefore \frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} = \frac{(\sqrt{8})^2 - \left( \sqrt{\frac{8}{7}} \right)^2}{(\sqrt{8})^2 + \left( \sqrt{\frac{8}{7}} \right)^2}$$

$$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}} = \frac{8 \left( 1 - \frac{1}{7} \right)}{8 \left( 1 + \frac{1}{7} \right)} = \frac{\frac{6}{7}}{\frac{8}{7}} = \frac{6}{8} = \frac{3}{4}$$

11. (b)  $\frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} = \frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} \cdot \frac{1 + \cos \alpha + \sin \alpha}{1 + \cos \alpha + \sin \alpha}$

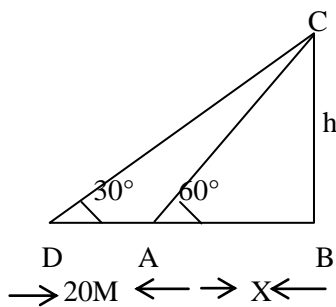
$$= \frac{(1 + \sin \alpha)^2 - \cos^2 \alpha}{(1 + \sin \alpha)(1 + \cos \alpha + \sin \alpha)}$$

$$= \frac{(1 + \sin^2 \alpha + 2 \sin \alpha) - (1 - \sin^2 \alpha)}{(1 + \sin \alpha)(1 + \cos \alpha + \sin \alpha)}$$

$$= \frac{2 \sin \alpha (1 + \sin \alpha)}{(1 + \sin \alpha)(1 + \cos \alpha + \sin \alpha)}$$

$$= \frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha} = y$$

12. (a) From right angled  $\Delta$ s ABC and DBC, we have



$$\tan 60^\circ = \frac{BC}{AB} \text{ and } \tan 30^\circ = \frac{BC}{DB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\text{and } \frac{1}{\sqrt{3}} = \frac{h}{x+20}$$

$$\Rightarrow h = x\sqrt{3}$$

$$\text{and } h = \frac{x+20}{\sqrt{3}}$$

$$\Rightarrow x\sqrt{3} = \frac{x+20}{\sqrt{3}} \Rightarrow 3x = x+20 \Rightarrow x = 10 \text{ m}$$

Putting  $x = 10$  in  $h = \sqrt{3}x$ , we get  $h = 10\sqrt{3}$   
Hence, height of the tree  $= 10\sqrt{3}$ m and the breadth of the river  $= 10$ m.

13. (a)  $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\tan^2\theta} = \frac{1}{8} = \frac{7}{8}$$

14. (a) Given,  $3 \cos\theta = 5 \sin\theta \Rightarrow \tan\theta = \frac{3}{5}$ .

$$\sec\theta = \sqrt{1 + \tan^2\theta} = \sqrt{1 + \left(\frac{3}{5}\right)^2} =$$

$$\sqrt{\frac{25+9}{25}} = \frac{\sqrt{34}}{5}$$

In expression, dividing the numerator & denominator by  $\cos\theta$ ,

$$\frac{5 \tan\theta - 2 \sec^4\theta + 2}{5 \tan\theta + 2 \sec^4\theta - 2}$$

$$= \frac{5 \times \frac{3}{5} - 2 \times \left(\frac{\sqrt{34}}{5}\right)^4 + 2}{5 \times \frac{3}{5} + 2 \times \left(\frac{\sqrt{34}}{5}\right)^4 - 2}$$

$$= \frac{3 - 2 \times \frac{1156}{625} + 2}{3 + 2 \times \frac{1156}{625} - 2} = \frac{5 - \frac{2312}{625}}{1 + \frac{2312}{625}}$$

$$= \frac{813}{2937} = \frac{271}{979}$$

15. (a) We have,  $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$  ... (i)

and  $x \sin\theta = y \cos\theta$  ... (ii)

Equation (1) may be written as

$$x \sin\theta \cdot \sin^2\theta + y \cos^3\theta = \sin\theta \cos\theta$$

$$\Rightarrow y \cos\theta \sin^2\theta + y \cos^3\theta = \sin\theta \cos\theta$$

$$\Rightarrow y \cos\theta \sin^2\theta + \cos^2\theta = \sin\theta \cos\theta$$

$$\Rightarrow y \cos\theta = \sin\theta \cos\theta$$

$$y = \sin\theta \quad \dots \text{(iii)}$$

Putting the value of  $y$  from (iii) in (ii), we get

$$x \sin\theta = \sin\theta \cdot \cos\theta \Rightarrow x = \cos\theta \quad \dots \text{(iv)}$$

Squaring (iii) and (iv), and adding, we get

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$$

16. (c) In the given equation,

$$1 + \sin^2 A = 3 \sin A \cos A$$

Dividing both sides by  $\cos^2 A$ ,

$$\text{We get } \frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} = 3 \cdot \frac{\sin A}{\cos A}$$

$$\Rightarrow \sec^2 A + \tan^2 A = 3 \tan A$$

$$\Rightarrow 1 + \tan^2 A + \tan^2 A = 3 \tan A$$

$$\Rightarrow 2 \tan^2 A - 3 \tan A + 1 = 0$$

$$\Rightarrow 2 \tan^2 A - 2 \tan A - \tan A + 1 = 0$$

$$\Rightarrow 2 \tan A (\tan A - 1) - 1(\tan A - 1) = 0$$

$$\Rightarrow (2 \tan A - 1)(\tan A - 1) = 0$$

$$\Rightarrow \tan A = \frac{1}{2}, 1$$

17. (c)  $\cos 20^\circ = \cos(90^\circ - 70^\circ) = \sin 70^\circ$

$$\cos 70^\circ = \sin 20^\circ$$

$$\therefore \frac{\cos^3 20^\circ - \cos^3 70^\circ}{\sin^3 70^\circ - \sin^3 20^\circ} = \frac{\sin^3 70^\circ - \sin^3 20^\circ}{\sin^3 70^\circ - \sin^3 20^\circ} = 1$$

18. (a)  $\frac{x \times 2^2 (\sqrt{2})^2}{8 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \left(\frac{\sqrt{3}}{2}\right)^2} = (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$$\text{or, } \frac{x \times 4 \times 2}{8 \times \frac{1}{2} \times \frac{3}{4}} = 3 - \frac{1}{3} \Rightarrow \frac{8x}{3} = \frac{9-1}{3}$$

$$\text{or, } \frac{8}{3}x = \frac{8}{3}$$

$$x = 1$$

19. (c) Given that  $\theta + \phi = \frac{\pi}{6}$

$$\Rightarrow \tan(\theta + \phi) = \tan \frac{\pi}{6}$$



$$\Rightarrow \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3} \tan \theta + \sqrt{3} \tan \phi = 1 - \tan \theta \tan \phi \quad \dots (1)$$

$$(\sqrt{3} + \tan \theta)(\sqrt{3} + \tan \phi)$$

$$= 3 + \sqrt{3} \tan \theta + \sqrt{3} \tan \phi + \tan \theta \tan \phi$$

$$= 3 + 1 - \tan \theta \tan \phi + \tan \theta \tan \phi = 4$$

20. (d)  $\sec^2 \theta \phi = 3 \Rightarrow \sec \theta = \sqrt{3}$

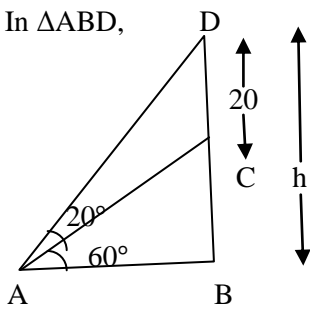
$$\tan^2 \theta = \sec^2 \theta - 1 = 3 - 1 = 2$$

$$\operatorname{cosec}^2 \theta = \frac{\sec^2 \theta}{\tan^2 \theta} = \frac{3}{2}$$

Now,  $\frac{\tan^2 \theta - \operatorname{cosec}^2 \theta}{\tan^2 \theta + \operatorname{cosec}^2 \theta} = \frac{2 - \frac{3}{2}}{2 + \frac{3}{2}}$

$$= \frac{\frac{1}{2}}{\frac{7}{2}} = \frac{1}{7}$$

21. (b) In  $\triangle ABD$ ,



$$\tan 60^\circ = \frac{BD}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{AB}$$

$$\Rightarrow AB = \frac{h}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{h}{3} \sqrt{3}$$

Now, in  $\triangle ABC$   
 $= AC^2 = AB^2 + BC^2$

$$\Rightarrow 20^2 = \left(\frac{h}{\sqrt{3}}\right)^2 + (h - 20)^2$$

$$\Rightarrow h^2 + 3h^2 - 120h = 0$$

$$\Rightarrow 4h^2 - 120h = 0$$

$$\Rightarrow h(h - 30) = 0$$

$$h = 0 \text{ or } 30$$

$h = 0$  not possible

$$\Rightarrow h = 30 \text{ ft}$$

22. (a)  $\sin \theta = \cos(2\theta - 45^\circ)$   
 or,  $\cos(90^\circ - \theta) = \cos(2\theta - 45^\circ)$   
 $\Rightarrow 90^\circ - \theta = 2\theta - 45^\circ$

$$\Rightarrow \theta = 45^\circ$$

$$\tan \theta \cdot \tan 45^\circ = 1$$

23. (b) Given,  $\sin 5\theta = \cos 4\theta = \sin(90^\circ - 4\theta)$

$$\Rightarrow 5\theta = 90^\circ - 4\theta$$

$$\theta = 10^\circ$$

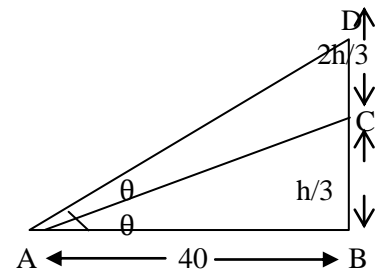
$$2 \sin 3\theta - \sqrt{3} \tan 3\theta$$

$$= 2 \times \frac{1}{2} - \sqrt{3} \times \frac{1}{\sqrt{3}} = 1 - 1 = 0.$$

24. (c) Let  $h$  be the height of pole, upper portion  $CD$  subtend angle  $\theta$  at  $A$ .

Then,  $\tan \theta = \frac{1}{2}$

Let lower part  $BC$  subtend angle  $\phi$  at  $A$  then  
 In  $\triangle ABC$ ,



$$\tan \phi = \frac{BC}{AB} = \frac{h/3}{40} = \frac{h}{120}$$

In  $\triangle ABD$ ,

$$\tan(\theta + \phi) = \frac{BD}{AB}$$

$$\Rightarrow \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi} = \frac{h}{40}$$

$$\Rightarrow \frac{\frac{1}{2} + \frac{h}{120}}{1 - \frac{h}{240}} = \frac{h}{40}$$

$$\Rightarrow \frac{2(60 + h)}{(240 - h)} = \frac{h}{40}$$

$$\Rightarrow 80(60 + h) = 240h - h^2 \Rightarrow 4800 + 80h = 240h - h^2$$

$$\Rightarrow h^2 - 160h + 4800 = 0$$

$$\Rightarrow (h - 120)(h - 40) = 0$$

$$\Rightarrow h = 120$$

[ $h = 40$  is discarded, since  $h > 100$  is given]

25. (b) Given,  $\sec \theta + \tan \theta = x \quad \dots (i)$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\Rightarrow (\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = 1$$



or  $\sec \theta - \tan \theta = \frac{1}{\sec \theta + \tan \theta} = \frac{1}{x}$  ... (ii)

Adding (i) & (ii), we get

$$2 \sec \theta = x + \frac{1}{x} = \frac{x^2 + 1}{x}$$

$$\sec \theta = \frac{x^2 + 1}{2x}$$

26. (d) Given identity

$$2(\sin^6 x + \cos^6 x) + t(\sin^4 x + \cos^4 x) = -1$$

$$\Rightarrow 2[(\sin^2 x + \cos^2 x)^3 - 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x) + t[(\sin^2 x + \cos^2 x)^2] - 2 \sin^2 x \cos^2 x] = -1$$

$[\because (a + b)^3 = a^3 + b^3 + 3ab(a + b)]$

and  $(a + b)^2 = a^2 + b^2 + 2ab$

where  $a = \sin^2 x, b = \cos^2 x$

$$\Rightarrow 2[1 - 3 \sin^2 x \cos^2 x] + t[1 - 2 \sin^2 x \cos^2 x] = -1$$

$$\Rightarrow 2 - 6 \sin^2 x \cos^2 x + t - 2t \sin^2 x \cos^2 x = -1$$

$$= t(1 - 2 \sin^2 x \cos^2 x) = -3(1 - 2 \sin^2 x \cos^2 x)$$

$\Rightarrow t = -3.$

27. (b)  $(a \cos \theta - b \sin \theta)^2 + (a \cos \theta + b \sin \theta)^2$

$$= a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \sin \theta \cos \theta + a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \cdot \sin \theta$$

$$= a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\sin^2 \theta + \cos^2 \theta)$$

$$= a^2 \times 1 + b^2 \times 1$$

$= a^2 + b^2.$

$\therefore (a \cos \theta - b \sin \theta)^2 + (a \cos \theta + b \sin \theta)^2 = a^2 + b^2.$

$$\Rightarrow c^2 + (a \cos \theta + b \sin \theta)^2 = a^2 + b^2$$

$$\Rightarrow a \cos \theta + b \sin \theta = \pm \sqrt{a^2 + b^2 - c^2}$$

28. (c)  $\tan \theta \left[ \frac{1}{\sec \theta - 1} + \frac{1}{\sec \theta + 1} \right]$

$$= \tan \theta \left[ \frac{\sec \theta + 1 + \sec \theta - 1}{(\sec \theta - 1)(\sec \theta + 1)} \right]$$

$$= \tan \theta \left[ \frac{2 \sec \theta}{\sec^2 \theta - 1} \right]$$

$$= \tan \theta \times \frac{2 \sec \theta}{\tan^2 \theta} = \frac{2 \sec \theta}{\tan \theta} = \frac{2}{\cos \theta \times \frac{\sin \theta}{\cos \theta}}$$

29. (d)  $(a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2 = m^2 + n^2$

$$a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \cdot \sin \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cdot \cos \theta = m^2 + n^2$$

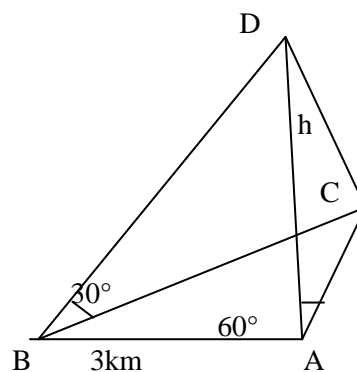
or  $a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\sin^2 \theta + \cos^2 \theta) = m^2 + n^2$

or  $a^2 + b^2 = m^2 + n^2$

30. In  $\Delta ACD$ , we get  $AC = h \cot 60^\circ = h \cdot (1/\sqrt{3})$ ,  
In  $\Delta BCD$ ,  $BC = h \cot 30^\circ = h\sqrt{3}$ .

Therefore, from right-angled triangle BAC, we have

$$BC^2 = AB^2 + AC^2$$



$$\Rightarrow (h\sqrt{3})^2 = (3)^2 + \left(\frac{h}{\sqrt{3}}\right)^2$$

$$\Rightarrow 3h^2 = 9 + \frac{h^2}{3} \Rightarrow \frac{8}{3}h^2 = 9$$

$$\Rightarrow h^2 = \frac{27}{8}$$

$$\Rightarrow h = \frac{3\sqrt{3}}{2\sqrt{2}} \text{ km} = \frac{3\sqrt{6}}{4} \text{ km}$$

31. (b)  $\sin \alpha + \cos \beta = 2$

$\sin \alpha \leq 1; \cos \beta \leq 2$

$\Rightarrow \alpha = 90^\circ; \beta = 0^\circ$

$\therefore \sin \left( \frac{2\alpha + \beta}{3} \right) = \sin \left( \frac{180^\circ}{3} \right)$

$= \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos \frac{\alpha}{3} = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$$32. \quad (c) \cos^4 \theta - \sin^4 \theta = \frac{2}{3}$$

$$\Rightarrow (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) = \frac{2}{3}$$

$$\Rightarrow \cos^2 \theta - \sin^2 \theta = \frac{2}{3}$$

$$\Rightarrow \cos^2 \theta - (1 - \cos^2 \theta) = \frac{2}{3}$$

$$\Rightarrow 2 \cos^2 \theta - 1 = \frac{2}{3}$$

$$33. \quad (a) \frac{\sin \alpha}{\cos(30^\circ + \alpha)} = 1$$

$$\Rightarrow \frac{\sin \alpha}{\sin(90^\circ - 30^\circ - \alpha)} = 1$$

$$\Rightarrow \frac{\sin \alpha}{\sin(60^\circ - \alpha)} = 1$$

$$\Rightarrow \sin \alpha = \sin(60^\circ - \alpha)$$

$$\Rightarrow \alpha = 60^\circ - \alpha$$

$$\Rightarrow 2\alpha = 60^\circ \Rightarrow \alpha = 30^\circ$$

$$\therefore \sin \alpha + \cos 2\alpha$$

$$= \sin 30^\circ + \cos 60^\circ$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$34. \quad (b) 2 \sin^2 \theta + 3 \cos^2 \theta$$

$$= 2 \sin^2 \theta + 2 \cos^2 \theta + \cos^2 \theta$$

$$= 2(\sin^2 \theta + \cos^2 \theta) + \cos^2 \theta$$

$$= 2 + \cos^2 \theta$$

$\therefore$  Minimum value of  $\cos \theta = -1$   
 $\therefore$  Required minimum value =  $2 + 1 = 3$

$$35. \quad (c) \frac{1}{\operatorname{cosec}^2 51^\circ} + \sin^2 39^\circ + \tan^2 51^\circ$$

$$= -1 \frac{1}{\sin^2 51^\circ \cdot \sec^2 39^\circ}$$

$$= \sin^2 51^\circ + \sin^2 39^\circ + \tan^2(90^\circ - 39^\circ)$$

$$= \sin^2 51^\circ + \sin^2 39^\circ + \cot^2 39^\circ$$

$$= \cos^2 39^\circ + \sin^2 39^\circ + \cot^2 39^\circ$$

$$= 1 + \cot^2 39^\circ - 1$$

$[\because \sin(90^\circ - \theta) = \cos \theta, \tan(90^\circ - \theta) = \cot \theta]$

$$= \operatorname{cosec}^2 39^\circ - 1 = x^2 - 1$$

$$36. \quad (b) \text{ When } \theta = 0^\circ$$

$$\sin^2 \theta + \cos^4 \theta = 1$$

When  $\theta = 45^\circ$ ,

$$\sin^2 \theta + \cos^4 \theta = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

When  $\theta = 30^\circ$

$$\sin^2 + \cos^4 \theta = \frac{1}{4} + \frac{9}{16} = \frac{13}{16}$$

$$37. \quad (c) \tan 2\theta = \frac{1}{\tan 4\theta} = \cot 4\theta$$

$$\Rightarrow \tan 2\theta = \tan(90^\circ - 4\theta)$$

$$\Rightarrow 2\theta = 90^\circ - 4\theta$$

$$\Rightarrow 6\theta = 90^\circ \Rightarrow \theta = 15^\circ$$

$$\therefore \tan 3\theta = \tan 45^\circ = 1$$

$$38. \quad (d) \tan(\theta_1 + \theta_2) = \sqrt{3} = \tan 60^\circ$$

$$\Rightarrow \theta_1 + \theta_2 = 60^\circ \text{ and } \sec(\theta_1 - \theta_2)$$

$$= \frac{2}{\sqrt{3}} = \sec 30^\circ$$

$$\Rightarrow \theta_1 - \theta_2 = 30^\circ$$

$$\therefore \theta_1 = 45^\circ \text{ and } \theta_2 = 15^\circ$$

$$\therefore \sin 2\theta_1 + \tan 3\theta_2$$

$$= \sin 90^\circ + \tan 45^\circ$$

$$= 1 + 1 = 2$$

$$39. \quad (b) \sec \theta = \frac{4x^2 + 1}{4x}$$

$$\tan \theta = \sqrt{\sec^2 \theta - 1}$$

$$= \sqrt{\left(\frac{4x^2 + 1}{4x}\right)^2 - 1}$$

$$= \sqrt{\frac{(4x^2 + 1)^2 - (4x)^2}{(4x)^2}}$$

$$= \frac{(2x + 1)(2x - 1)}{4x} = \frac{4x^2 - 1}{4x}$$

$$\therefore \sec \theta + \tan \theta = \frac{4x^2 + 1}{4x} + \frac{4x^2 - 1}{4x}$$

$$= \frac{4x^2 + 1 + 4x^2 - 1}{4x}$$

$$= \frac{8x^2}{4x} = 2x$$

$$40. \quad (a)$$

$$x = a \sec \theta \cdot \cos \phi; y = b \sec \theta \cdot \sin \phi; z = c \tan \theta$$

$$\begin{aligned} & \therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} \\ & = \frac{a^2 \sec^2 \theta \cos^2 \phi}{a^2} + \frac{b^2 \sec^2 \theta \sin^2 \phi}{b^2} \\ & \quad - \frac{c^2 \tan^2 \theta}{c^2} \\ & = \sec^2 \theta \cdot \cos^2 \phi + \sec^2 \theta \cdot \sin^2 \phi - \tan^2 \theta \\ & = \sec^2 \theta (\cos^2 \phi + \sin^2 \phi) - \tan^2 \theta \\ & = \sec^2 \theta - \tan^2 \theta \end{aligned}$$

41. (a)  $\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = \frac{5}{3}$   
 $\Rightarrow 5 \sec \theta - 5 \tan \theta = 3 \sec \theta + 3 \tan \theta$   
 $\Rightarrow 2 \sec \theta = 8 \tan \theta$   
 $\Rightarrow \frac{\tan \theta}{\sec \theta} = \frac{2}{8} = \frac{1}{4}$   
 $\Rightarrow \frac{\sin \theta}{\cos \theta} \times \cos \theta = \frac{1}{4}$   
 $\Rightarrow \sin \theta = \frac{1}{4}$

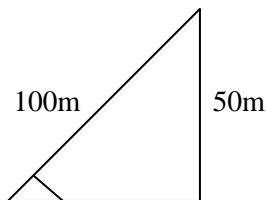
42. (b)  $\sec^2 \theta + \tan^2 \theta = 7$   
 $1 + \tan^2 + \tan^2 \theta = 7$   
 $(\because 1 + \tan^2 \theta = \sec^2 \theta)$   
 $\tan^2 \theta = \frac{6}{2} = 3$   
 $\tan \theta = \pm \sqrt{3}$   
 $\tan \theta = \sqrt{3}$  (or)  
 $\tan \theta = -\sqrt{3}$

As  $0 \leq \theta \leq \pi/2$

$$\begin{aligned} \therefore \theta &= \tan^{-1} \sqrt{3} \\ \theta &= \frac{\pi}{3} \end{aligned}$$

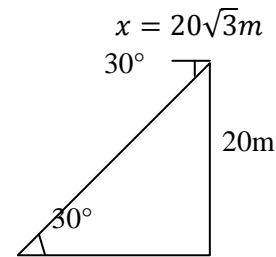
43. (c)  $\sin^2 x + 2 \tan^2 \theta - 2 \sec^2 x + \cos^2 x$   
 $\sin^2 x + \cos^2 x - 2(\sec^2 x - \tan^2 x)$   
 $1 - 2(1) = -1$

44. (d)



$$\begin{aligned} \sin \theta &= \frac{50m}{100m} = \frac{1}{2} \\ \theta &= 30^\circ \end{aligned}$$

45. (b)  $\tan 30^\circ = \frac{20}{x}$   
 $\frac{1}{\sqrt{3}} = \frac{20}{x}$



46. (a)  $\frac{a^2}{x^2} - \frac{b^2}{y^2} = \frac{a^2}{a^2 \sin^2 \theta} - \frac{b^2}{b^2 \tan^2 \theta}$   
 $\Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

47. (b)  $2y \cos \theta = x \sin \theta$   
 $\Rightarrow \sin \theta = \frac{2y}{x} \cos \theta$   
 And  $2x \sec \theta - y \operatorname{cosec} \theta = 3$   
 $\Rightarrow 2x \sec \theta - \frac{y}{\sin \theta} = 3$   
 $\Rightarrow \frac{2x}{\cos \theta} - \frac{yx}{2y \cos \theta} = 3$   
 $\Rightarrow 3 \cos \theta = \frac{3}{2} x \Rightarrow \cos \theta = \frac{x}{2}$

Now  $\sin^2 \theta + \cos^2 \theta = 1$   
 $= y^2 + \frac{x^2}{4} = 1$   
 $\Rightarrow 4y^2 + x^2 = 4$

48. (b)  $\sec^2 \theta - \tan^2 \theta = 1$   
 $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$   
 $\sqrt{3}(\sec \theta - \tan \theta) = 1 \Rightarrow \sec \theta - \tan \theta = \frac{1}{\sqrt{3}}$  ... (1)

$\sec \theta + \tan \theta = \sqrt{3}$  (Given) ... (2)

Adding eqns (1) and (2)

$$\begin{aligned} 2 \sec \theta &= \sqrt{3} + \frac{1}{\sqrt{3}} \Rightarrow 2 \sec \theta = \frac{4}{\sqrt{3}} \Rightarrow \sec \theta \\ &= \frac{2}{\sqrt{3}} \\ \therefore \cos \theta &= \frac{\sqrt{3}}{2} \left[ \because \sec \theta = \frac{1}{\cos \theta} \right] \end{aligned}$$

Therefore,  $\sin \theta = \sqrt{1 - \cos^2 \theta}$

$$\Rightarrow \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$$

49. (a)  $63^\circ 14' \left( \frac{51}{60} \right)$  [1 minute = 60 seconds]

$$\Rightarrow 63^{\circ} \left[ 14 + \frac{17}{20} \right] \Rightarrow 63^{\circ} \left[ \frac{297}{20} \right]$$

$$\Rightarrow 63^{\circ} + \frac{297}{20 \times 60}$$

[1 degree = 60 minutes]

$$\Rightarrow \left( \frac{75897}{1200} \right)^{\circ} \Rightarrow \frac{75897}{1200} \times \frac{\pi}{180} \text{radian}$$

$$\Rightarrow \left( \frac{2811}{8000} \right)^{\circ}$$

50. (d)  $\frac{\cos^4 \alpha}{\cos^2 \beta} + \frac{\sin^4 \alpha}{\sin^2 \beta} = 1$

$$\Rightarrow \cos^4 \alpha \sin^2 \beta + \sin^4 \alpha \cos^2 \beta = \cos^2 \beta \sin^2 \beta$$

$$\Rightarrow \cos^4 \alpha (1 - \cos^2 \beta) + \cos^2 \beta (1 - \cos^2 \alpha)^2 = \cos^2 \beta \sin^2 \beta$$

$$\Rightarrow \cos^4 \alpha - \cos^4 \alpha \cos^2 \beta + \cos^2 \beta - 2 \cos^2 \alpha \cos^2 \beta + \cos^4 \alpha \cos^2 \beta = \cos^2 \beta \sin^2 \beta$$

$$\Rightarrow \cos^4 \alpha - 2 \cos^2 \alpha \cos^2 \beta + \cos^4 \beta = 0$$

$$\Rightarrow (\cos^2 \alpha - \cos^2 \beta)^2 = 0$$

$$\Rightarrow \cos^2 \alpha = \cos^2 \beta$$

$$\Rightarrow \sin^2 \alpha = \sin^2 \beta$$

Then,  $\frac{\cos^4 \beta}{\cos^2 \alpha} + \frac{\sin^2 \beta}{\sin^2 \alpha}$

51. (a)  $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1}$

Dividing Numerator and Denominator by  $\cos \theta$

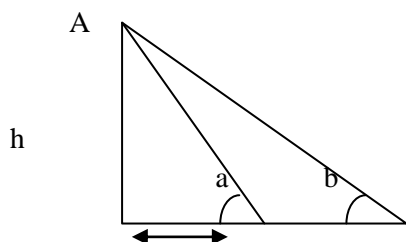
$$\Rightarrow \frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\cos \theta} + \frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\cos \theta} - \frac{1}{\cos \theta}} = \frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta}$$

$$\Rightarrow \frac{(\tan \theta + \sec \theta) - (\sec^2 \theta - \tan^2 \theta)}{\tan \theta - \sec \theta + 1}$$

$$\Rightarrow \frac{(\tan \theta + \sec \theta) - [1 - \sec \theta + \tan \theta]}{\tan \theta - \sec \theta + 1}$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} = \frac{1 + \sin \theta}{\cos \theta}$$

52. (b) In  $\triangle ABC$



B 9 C D

$$\tan \alpha = \frac{h}{9} \dots (1)$$

In  $\triangle ABD$

$$\tan \beta = \frac{h}{16}$$

$$\alpha + \beta = 90^{\circ} (\text{given})$$

$$\beta = 90 - \alpha$$

since  $\tan \beta = \frac{h}{16}$

$$\tan(90 - \alpha) = \frac{h}{16} \Rightarrow \cot \alpha = \frac{h}{16} \text{ or } \tan \alpha = \frac{16}{h} \dots (2)$$

From eqn. (1) and (2)

$$\frac{h}{9} = \frac{16}{h} \Rightarrow h^2 = 16 \times 9 \Rightarrow h = 12 \text{ feet.}$$

53. (a) If  $\sin^2 \alpha = \cos^3 \alpha$

$$\tan^2 \alpha = \cos \alpha \dots (1)$$

Now, consider,  $\cot^6 \alpha - \cot^2 \alpha$

$$= \frac{1}{\tan^6 \alpha} - \frac{1}{\tan^2 \alpha} \text{ Since } \cot \alpha = \frac{1}{\tan \alpha}$$

Substituting for  $\tan^2 \alpha$  with  $\cos \alpha$  from (1) above equation will be

$$= \frac{1}{\cos^3 \alpha} - \frac{1}{\cos \alpha} = \frac{1 - \cos^2 \alpha}{\cos^3 \alpha} = \frac{\sin^2 \alpha}{\cos^3 \alpha}$$

$$= \frac{\tan^2 \alpha}{\cos \alpha} = 1$$

54. (b)  $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$

$$\Rightarrow \left( 1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} \right) \left( 1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} \right)$$

$$\Rightarrow \left( \frac{\sin \theta + \cos \theta + 1}{\cos \theta} \right) \left( \frac{\sin \theta + \cos \theta - 1}{\sin \theta} \right)$$

$$= \frac{(\sin \theta + \cos \theta)^2 - 1}{\sin \theta \cos \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2$$

55. (b)  $\frac{\sin 53^{\circ}}{\cos 37^{\circ}} + \frac{\cot 65^{\circ}}{\tan 25^{\circ}}$

$$\frac{\sin 53^{\circ}}{\cos 37^{\circ}} \times \frac{\tan 25^{\circ}}{\cot 65^{\circ}}$$

$$\Rightarrow \frac{\sin 53^{\circ}}{\cos(90^{\circ} - 53^{\circ})}$$

$$\times \frac{\tan 25^{\circ}}{\cot(90^{\circ} - 25^{\circ})}$$

$$\Rightarrow \frac{\sin 53^\circ}{\sin 53^\circ} \times \frac{\tan 25^\circ}{\tan 25^\circ} = 1$$

[∴  $\cos(90^\circ - \theta) = \sin \theta$  and  $\cot(90^\circ - \theta) = \tan \theta$ ]

56. (c)  $\frac{\cos 60^\circ + \sin 60^\circ}{\cos 60^\circ - \sin 60^\circ} = \frac{\frac{1}{2} + \frac{\sqrt{3}}{2}}{\frac{1}{2} - \frac{\sqrt{3}}{2}} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$

$$\Rightarrow \frac{(1 + \sqrt{3})^2}{1^2 - (\sqrt{3})^2} = \frac{1 + 3 + 2\sqrt{3}}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2}$$

$$\Rightarrow \frac{-2(2 + \sqrt{3})}{2} = -(2 + \sqrt{3})$$

57. (d)  $\frac{\cot 5^\circ \cdot \cot 10^\circ \cdot \cot 15^\circ \cdot \cot 60^\circ \cdot \cot 75^\circ \cdot \cot 80^\circ \cdot \cot 85^\circ}{(\cos^2 20^\circ + \cos^2 70^\circ) + 2}$

$$\frac{\cot(90^\circ - 85^\circ) \cdot \cot(90^\circ - 80^\circ) \cdot \cot(90^\circ - 75^\circ) \cdot \cot 60^\circ \cdot \cot 75^\circ \cdot \cot 80^\circ \cdot \cot 85^\circ}{(\cos^2(90^\circ - 70^\circ) + \cos^2 70^\circ) + 2}$$

$$\Rightarrow \frac{\cot 60^\circ}{(1 + 2)} = \frac{1}{3} = \frac{1}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{9}$$

58. (d) Let angles are  $2x$ ,  $5x$  and  $3x$ .  
 $2x + 5x + 3x = 180^\circ$   
 (sum of integer angle of triangles is  $180^\circ$ )  
 $10x = 180^\circ$   
 $x = 18^\circ$   
 ∴ Least angle in degree =  $2x = 2 \times 18 = 36^\circ$   
 In radian =  $\frac{\pi}{180^\circ} \times 36^\circ = \frac{\pi}{5}$

59. (d)  $x = a \cos \theta - b \sin \theta$   
 $y = b \cos \theta + a \sin \theta$   
 $x^2 + y^2 = (a \cos \theta - b \sin \theta)^2 + (b \cos \theta + a \sin \theta)^2$

$$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \cos \theta \sin \theta + b^2 \cos^2 \theta + a^2 \sin^2 \theta + 2ab \cos \theta \sin \theta$$

$$\Rightarrow (a^2 + b^2) \cos^2 \theta + (a^2 + b^2) \sin^2 \theta$$

$$\Rightarrow a^2 + b^2(\cos^2 \theta + \sin^2 \theta)$$

$$\Rightarrow a^2 + b^2(1) \Rightarrow a^2 + b^2$$

60. (a)  $\tan \alpha + \cot \alpha = 2$

$$\tan \alpha + \frac{1}{\tan \alpha} = 2 \Rightarrow \tan^2 \alpha + 1 = 2 \tan \alpha$$

$$\Rightarrow \tan^2 \alpha - 2 \tan \alpha + 1 = 0$$

$$\Rightarrow \tan^2 \alpha - \tan \alpha - \tan \alpha + 1 = 0$$

$$\Rightarrow \tan \alpha (\tan \alpha - 1) - 1(\tan \alpha - 1) = 0$$

$$(\tan \alpha - 1)(\tan \alpha - 1) = 0$$

$$\therefore \tan \alpha = 1$$

Now,  $\tan^7 \alpha + \cot^7 \alpha \Rightarrow (\tan \alpha)^7 + \frac{1}{(\tan \alpha)^7} = 1 + 1 = 2$

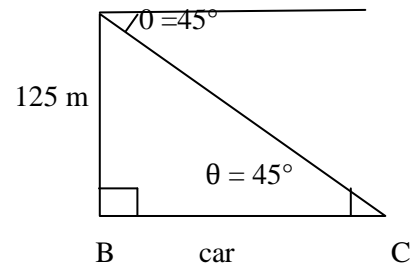
$$\therefore \tan \alpha = 1$$

Now,  $\tan^7 \alpha + \cot^7 \alpha \Rightarrow (\tan \alpha)^7 + \frac{1}{(\tan \alpha)^7} =$

$$1 + 1 = 2$$

(a) Tower

A



In  $\triangle ABC$

$$\tan \theta = \frac{AB}{BC} \Rightarrow \tan 45^\circ = \frac{125}{BC} \Rightarrow 1 = \frac{125}{BC}$$

$BC = 125\text{m}$

Hence, car is 125 m from the tower.

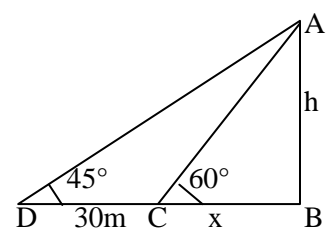
62. (c)  $\frac{(\cos \theta + \sin \theta)(\cos^2 \theta + \sin^2 \theta + \sin \theta \cos \theta)}{(\cos \theta + \sin \theta)}$

$$+ \frac{(\cos \theta - \sin \theta)(\cos^2 \theta + \sin^2 \theta + \sin \theta \cos \theta)}{(\cos \theta - \sin \theta)}$$

$$= 2 \cos^2 \theta + 2 \sin^2 \theta - \sin \theta \cos \theta + \sin \theta \cos \theta$$

$$= 2$$

63. (a)



In  $\triangle ABC$ ,  $\tan 60^\circ = \frac{h}{x}$

$$x = \frac{h}{\sqrt{3}}$$

In  $\triangle ABD$ ,  $\tan 45^\circ = \frac{h}{30+x}$

$$1 = \frac{h}{30+x} \text{ or } h = 30 + x$$

Putting value of  $x$  from (1)

$$h = 30 + \frac{h}{\sqrt{3}}$$

$$\text{or } h \frac{(\sqrt{3}-1)}{\sqrt{3}} = 30 \Rightarrow h = 15(3 + \sqrt{3})\text{m}$$

64. (d)  $\sin 17^\circ = \frac{x}{y}$

$$\cos 17^\circ = \sqrt{1 - \frac{x^2}{y^2}} = \frac{\sqrt{y^2 - x^2}}{y}$$

$$\begin{aligned} \sec 17^\circ - \sin 73^\circ &= \sec 17^\circ - \cos 17^\circ \\ &= \frac{y}{\sqrt{y^2 - x^2}} - \frac{\sqrt{y^2 - x^2}}{y} \\ &= \frac{y^2 - y^2 + x^2}{y\sqrt{y^2 - x^2}} = \frac{x^2}{y\sqrt{y^2 - x^2}} \end{aligned}$$

65. (c)  $\operatorname{cosec} \theta + \cot \theta = \sqrt{3}$

$$\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = \sqrt{3}$$

$$\frac{1 + \cos \theta}{\sin \theta} = \sqrt{3}$$

$$\frac{2 \cos^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} = \sqrt{3}$$

$$\cot \frac{\theta}{2} = \sqrt{3}$$

$$\tan \frac{\theta}{2} = \frac{1}{\sqrt{3}}; \frac{\theta}{2} = 30^\circ; \theta = 60^\circ$$

66. (c)  $\cos \alpha + \sec \alpha = \sqrt{3}$   
 talking cube both sides

$$\begin{aligned} \cos^3 \alpha + \sec^3 \alpha &= \cos^3 \alpha + 3 \cos \alpha \sec \alpha (\cos \alpha \\ &+ \sec \alpha) + \sec^3 \alpha = 3\sqrt{3} \\ \cos^3 \alpha + \sec^3 \alpha &= 0 \end{aligned}$$

67. (a)  $\sin \theta + \cos \theta = \sqrt{2} \cos \theta$

$$\sin \theta = (\sqrt{2} - 1) \cos \theta$$

$$\cot \theta = \frac{1}{\sqrt{2} - 1}$$

$$\cot \theta = \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \sqrt{2} + 1$$

68. (d)  $(\sin^2 1^\circ + \sin^2 89^\circ) + (\sin^2 2^\circ + \sin^2 88^\circ) + \dots + (\sin^2 44^\circ + \sin^2 48^\circ) + \sin^2 45^\circ$

$$\begin{aligned} &= (\sin^2 1^\circ + \cos^2 1^\circ) + (\sin^2 2^\circ + \cos^2 2^\circ) \\ &+ \dots + (\sin^2 44^\circ + \cos^2 44^\circ) \\ &+ \sin^2 45^\circ \\ &= 1 + 1 + \dots + 1(44 \text{ times}) + \frac{1}{2} \\ &= 44 \frac{1}{2} \end{aligned}$$

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